TIME RESPONSE

Chapter 4

EE3302

Lecture abstract

- Topics covered in this presentation
- Poles & zeros
- First-order systems
- Second-order systems
- Effect of additional poles
- Effect of zeros

Chapter outline

4 Time response

- 4.1 Introduction
- 4.2 Poles, zeros, and system response
- 4.3 First-order systems
- 4.4 Second-order systems: introduction
- 4.5 The general second-order system
- 4.6 Underdamped second-order systems
- 4.7 System response with additional poles
- 4.8 System response with zeros
- 4.9 Effects of nonlinearities upon time responses
- 4.10 Laplace transform solution of state equations
- 4.11 Time domain solution of state equations

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Definitions

Poles of a TF

- Values of the Laplace transform variable, ***s***, that cause the TF to become infinite
- Any roots of the denominator of the TF that are common to the roots of the numerator

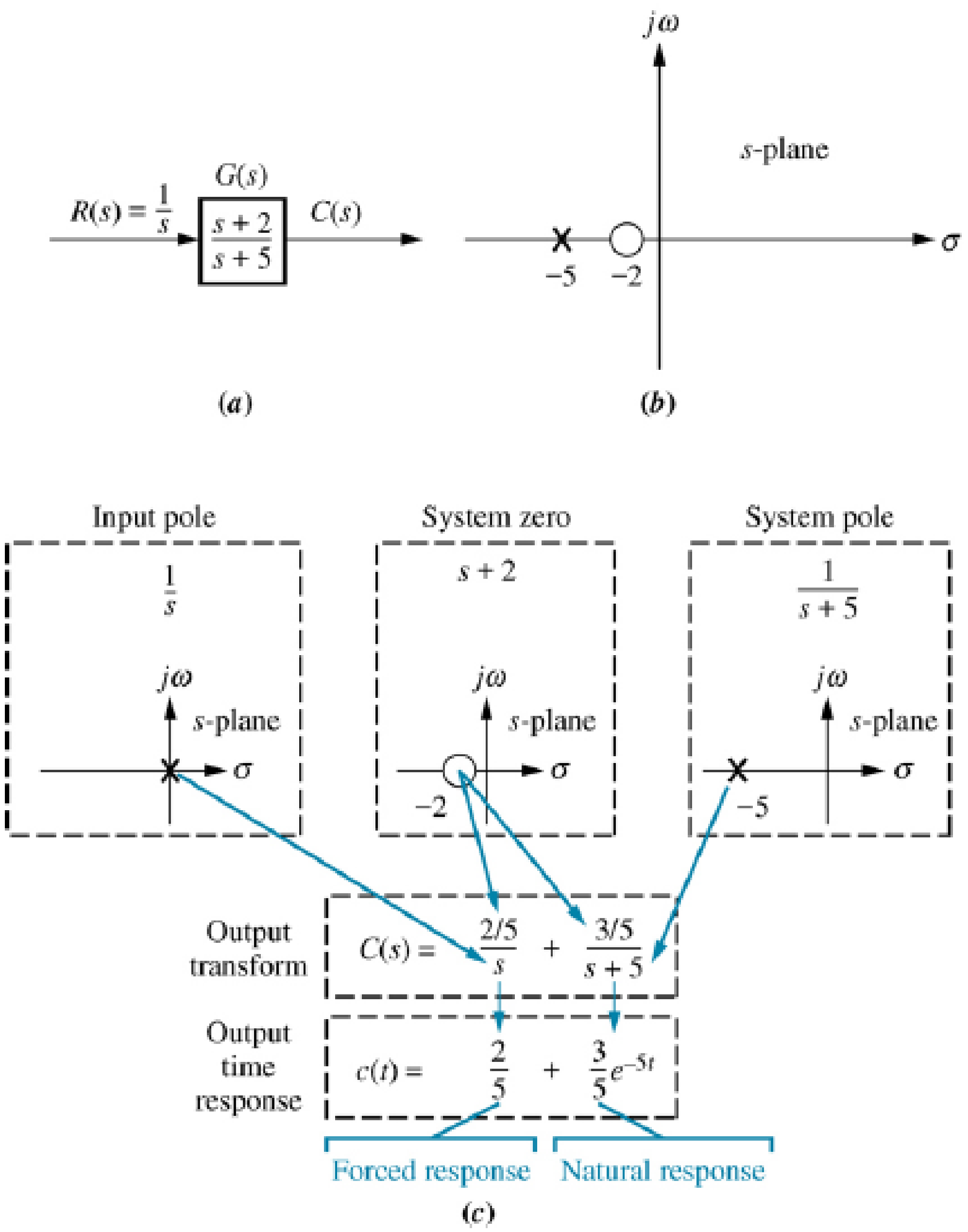

Figure: a. system showing input & output, b. pole-zero plot of the system; c. evolution of a system response

Definitions

Zeros of a TF

- Values of the Laplace transform variable, s, that cause the TF to become zero
- Any roots of the numerator of the TF that are common to the roots of the denominator

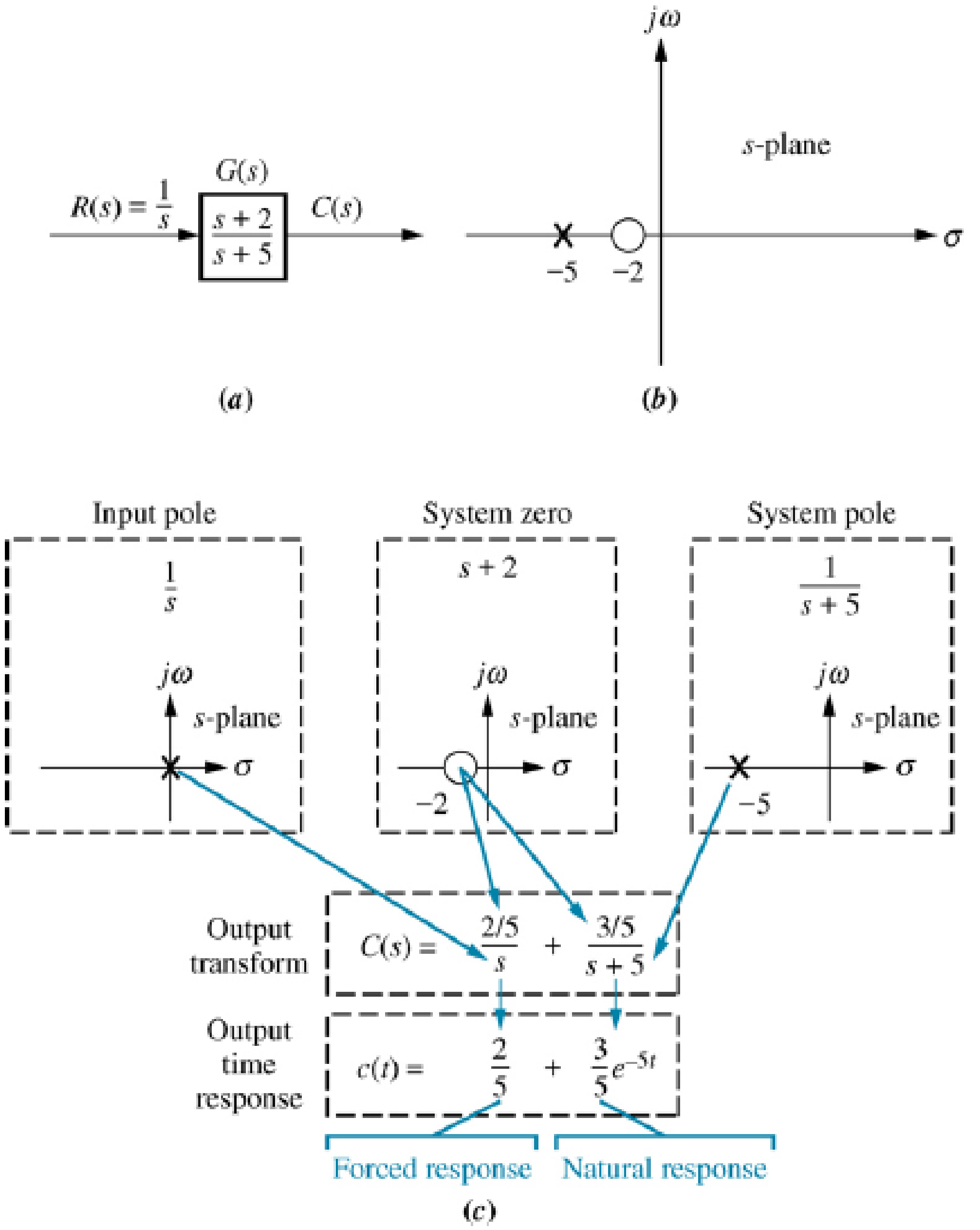

Figure: a. system showing input & output, b. pole-zero plot of the system; c. evolution of a system response

System response characteristics

Poles of a TF:

- Generate the form of the natural response

Poles of an input function:

- Generate the form of the forced response

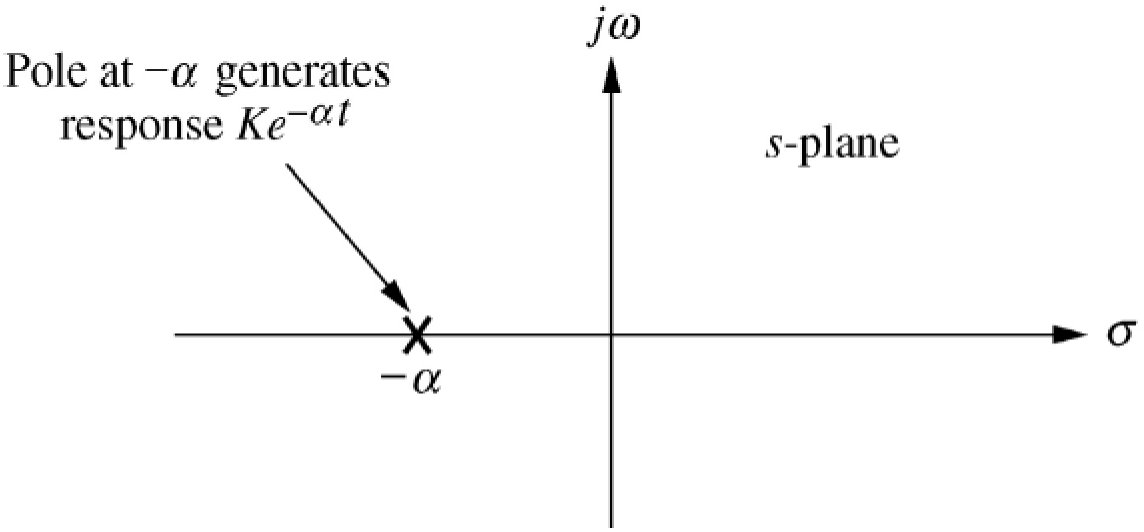

Figure: Effect of a real-axis pole upon transient response

System response characteristics

Pole on the real axis:

- Generates an exponential response of the form $e^{-\alpha t}$, where $-\alpha$ is the pole location on the real axis. The farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero.

Zeros and poles:

- Generate the amplitudes for both the forced and natural responses

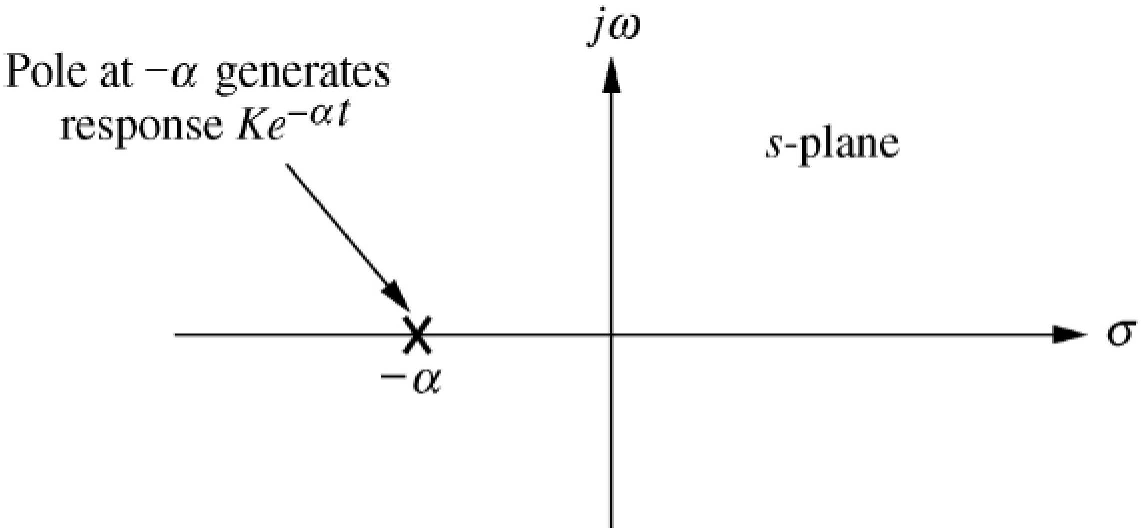

Figure: Effect of a real-axis pole upon transient response

Example 4.1

Evaluating Response Using Poles

PROBLEM: Given the system of Figure 4.3, write the output, $c(t)$, in general terms. Specify the forced and natural parts of the solution.

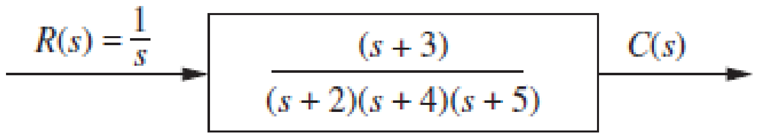

FIGURE 4.3 System for Example 4.1

SOLUTION: By inspection, each system pole generates an exponential as part of the natural response. The input's pole generates the forced response. Thus,

$$C(s) \equiv \underbrace{\frac{K_1}{s}}_{\text{Forced response}} + \underbrace{\frac{K_2}{s+2} + \frac{K_3}{s+4} + \frac{K_4}{s+5}}_{\text{Natural response}} \tag{4.3}$$

Taking the inverse Laplace transform, we get

$$c(t) \equiv \underbrace{K_1}_{\text{Forced response}} + \underbrace{K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}}_{\text{Natural response}} \tag{4.4}$$

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Intro

- 1st-order system without zeros TF

$$G(s) = \frac{C(s)}{R(s)} = \frac{a}{s+a}$$

- Unit step input TF $R(s) = s^{-1}$
- System response in frequency domain

$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

- System response in time domain

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

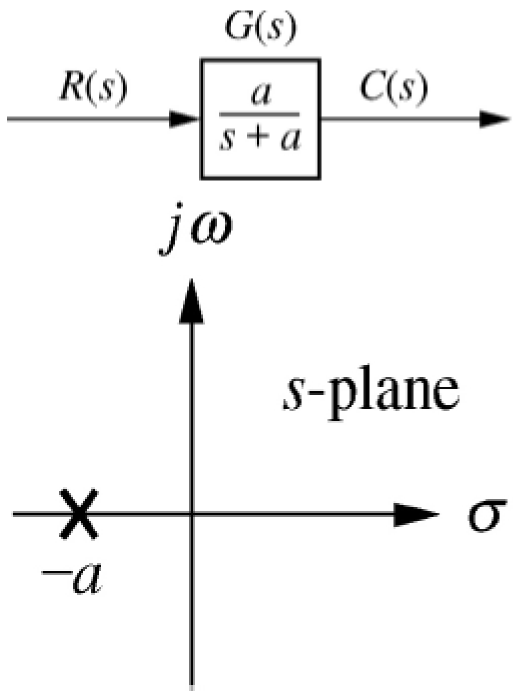

Figure: 1st-order system; pole-plot

Characteristics

- ***Time constant, 1/a :***

➢The time for e^{-at} to decay to 37% of its initial value. Alternatively, the time it takes for the step response to rise to 63% of its final value.

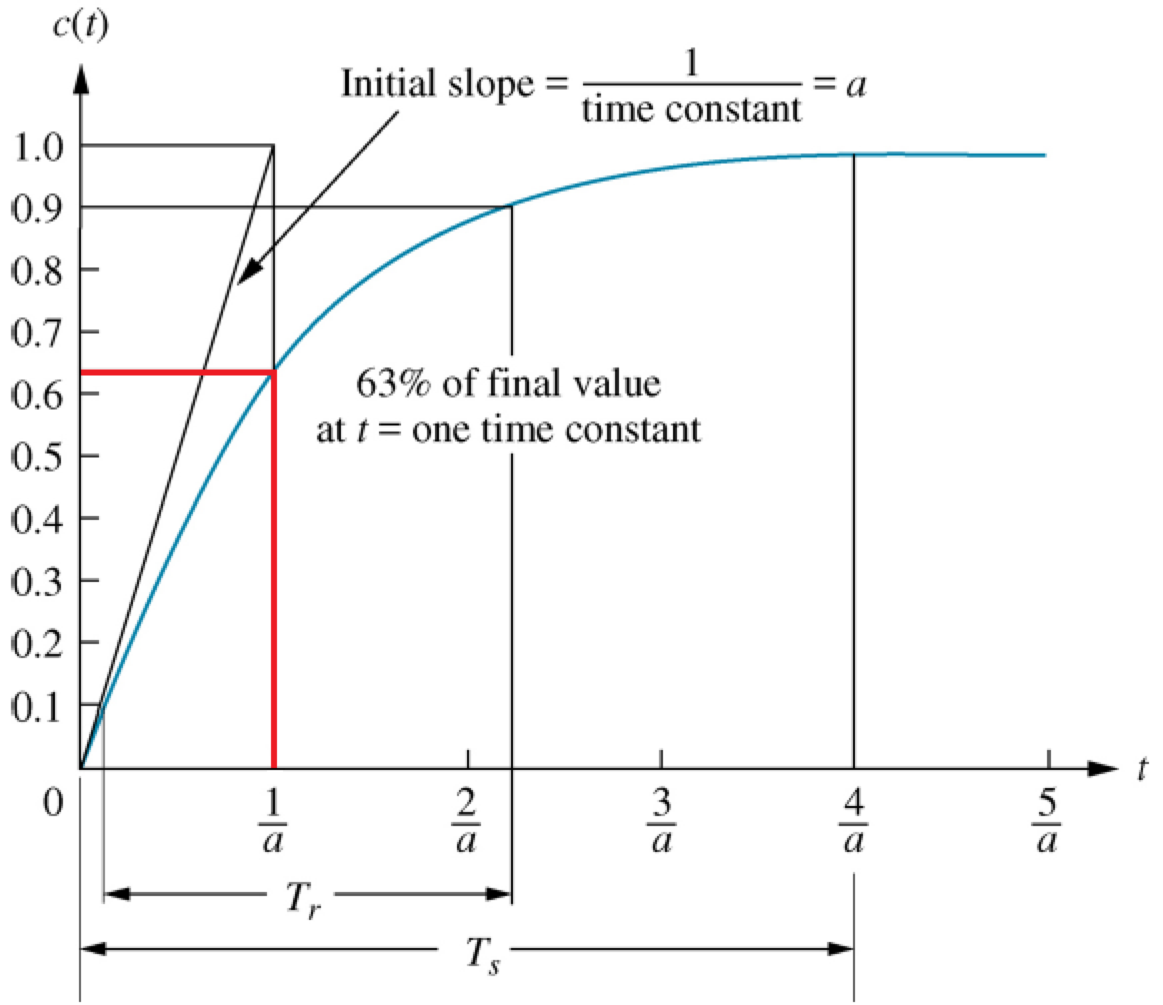

Figure: 1st-order system response to a unit step

Characteristics

- ***Exponential frequency, a:***

➢ The reciprocal of the time constant. The initial rate of change of the exponential at t=0, since the derivative of e^{-at} is −a when t = 0.

➢ Since the pole of the TF is at −a, the farther the pole is from the imaginary axis, the faster the transient response.

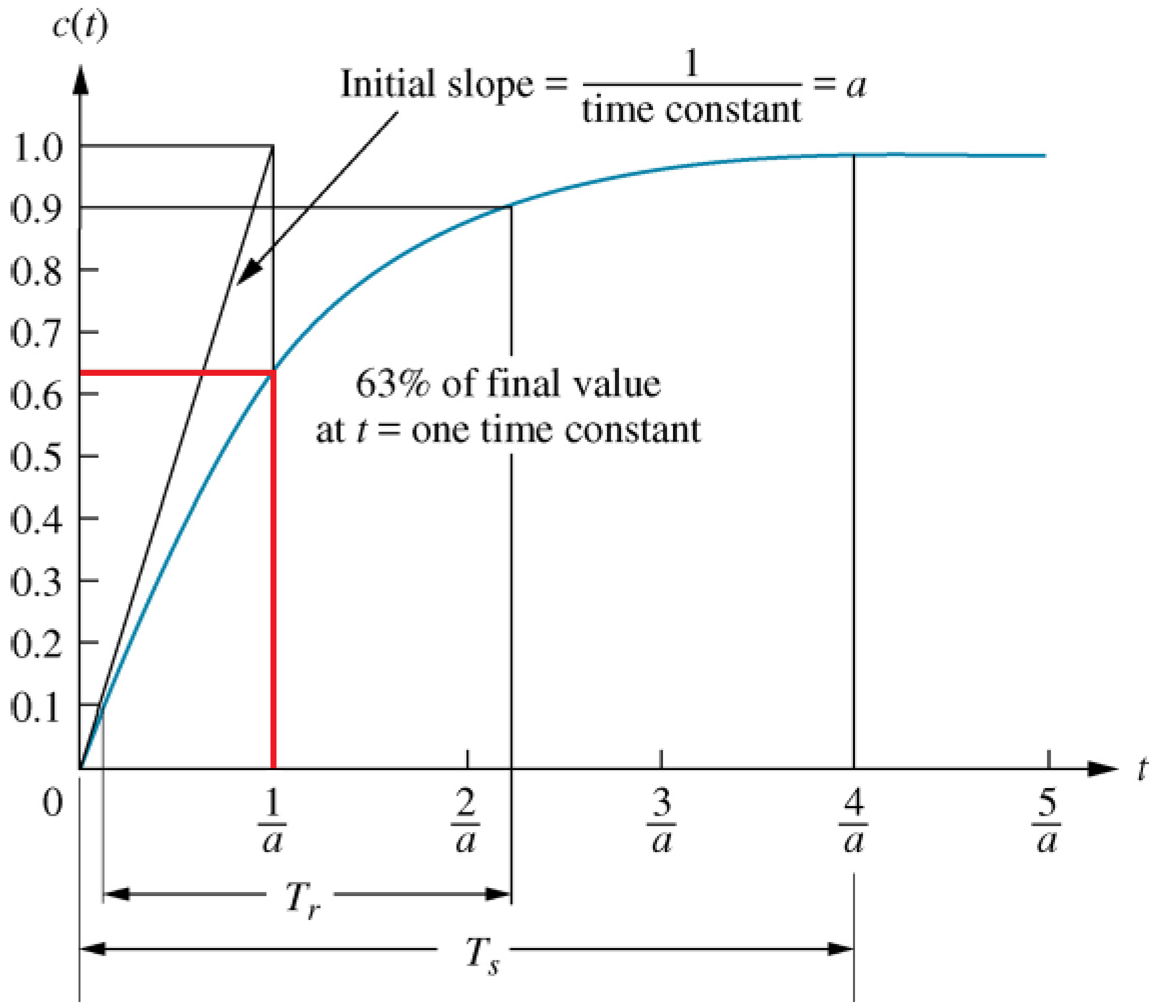

Figure: 1st-order system response to a unit step

Characteristics

- ***Rise time, T_r:***

➢The time for the waveform to go from 0.1 to 0.9 of its final value. The difference in time between *c(t) = 0.9* and *c(t) = 0.1*.

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

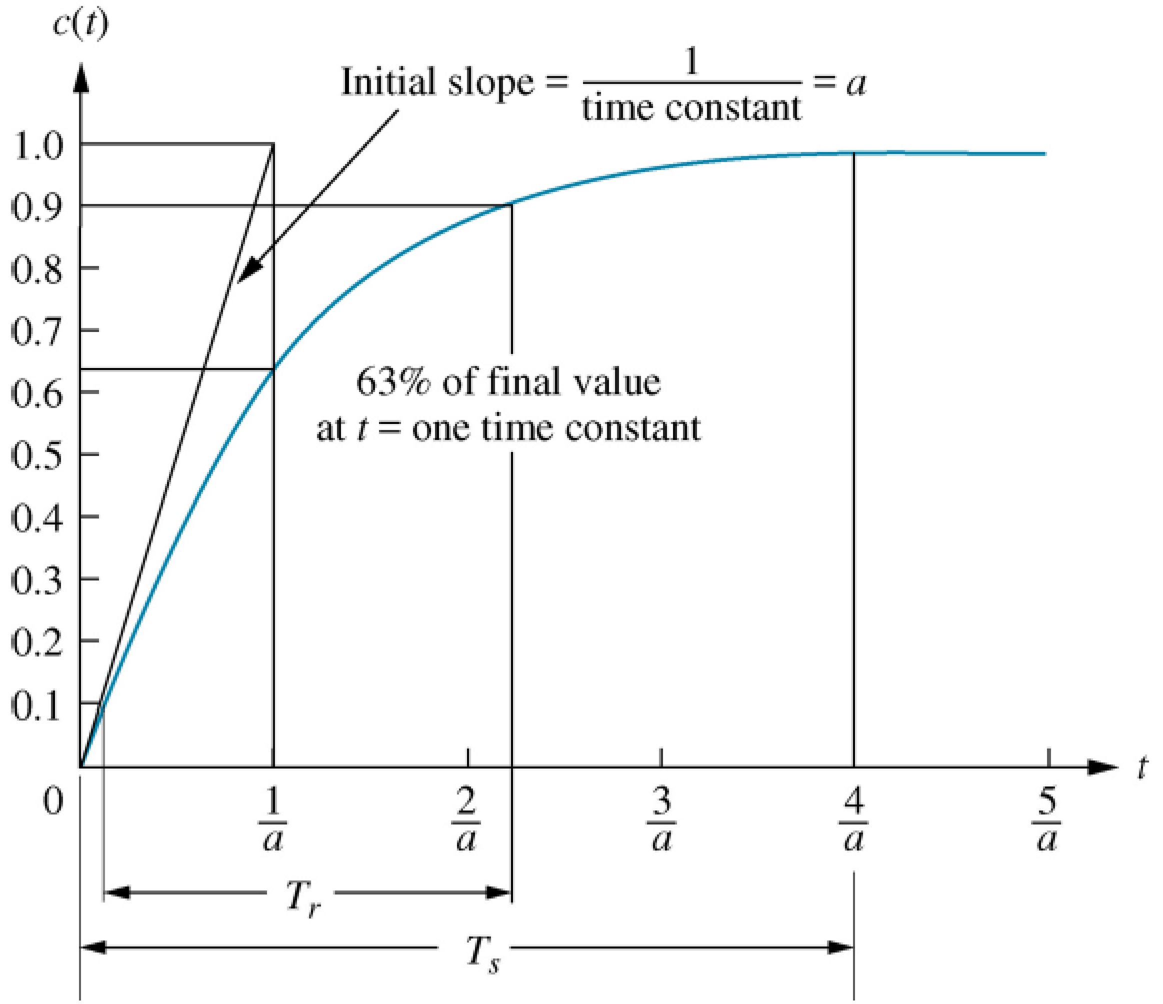

Figure: 1st-order system response to a unit step

Characteristics

- ***2% Settling time, T_s:***

➢The time for the response to reach, and stay within, 2% (arbitrary) of its final value. The time when *c(t) = 0.98*.

$$T_s = \frac{4}{a}$$

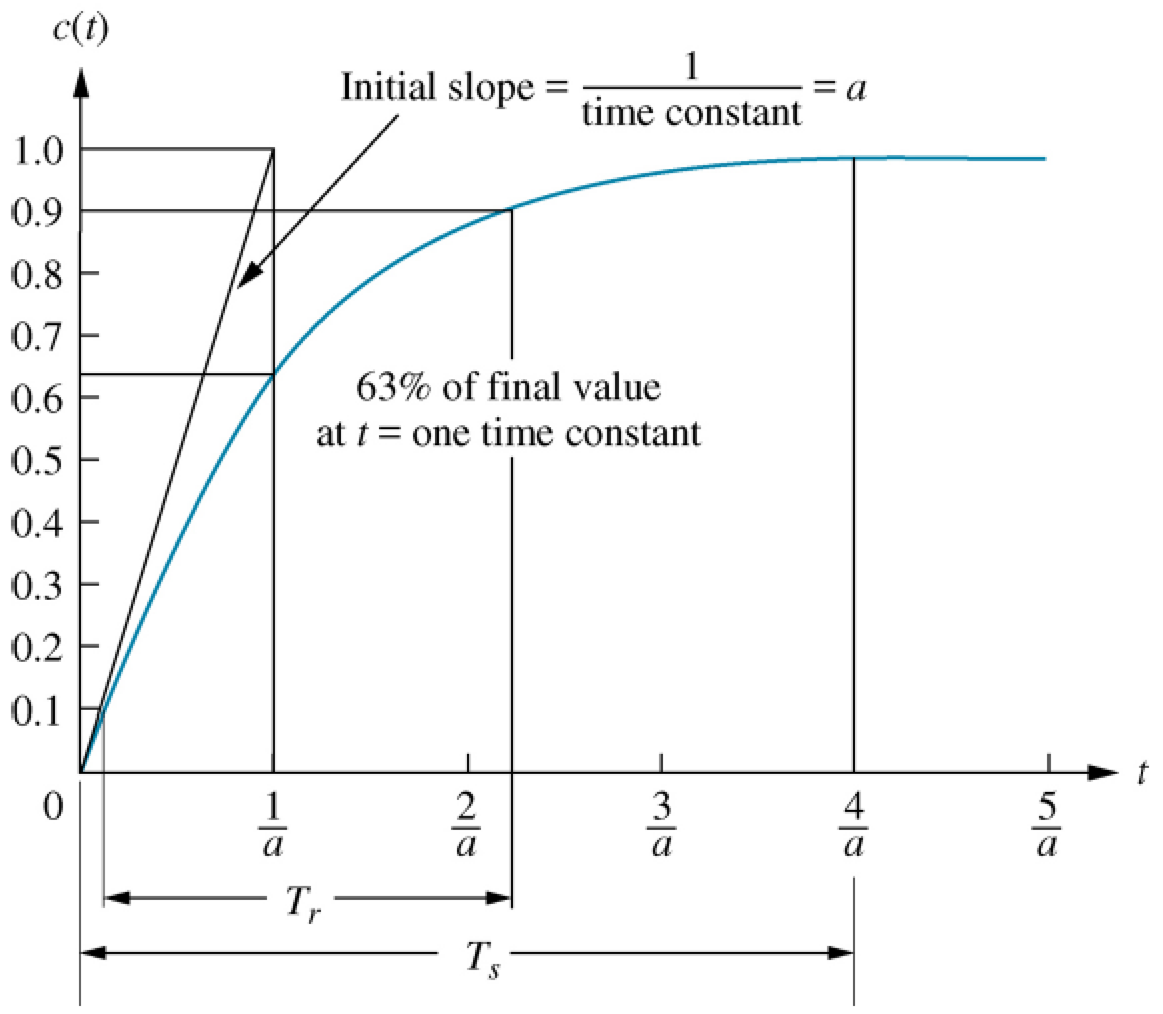

Figure: 1st-order system response to a unit step

First-Order Transfer Functions via Testing

- Consider a simple first-order system, $G(s)=K/(s + a)$, whose step response is

$$C(s) = \frac{K}{s(s+a)} = \frac{K/a}{s} - \frac{K/a}{(s+a)}$$

- we can identify K and a from laboratory testing, so we can obtain the transfer function of the system.

First-Order Transfer Functions via Testing

- We determine that it has the first-order characteristics we have seen thus far, such as:
  - no overshoot.
  - nonzero initial slope.
- The transfer function for the system is

$$G(s) = 5.54/(s+7.7)$$

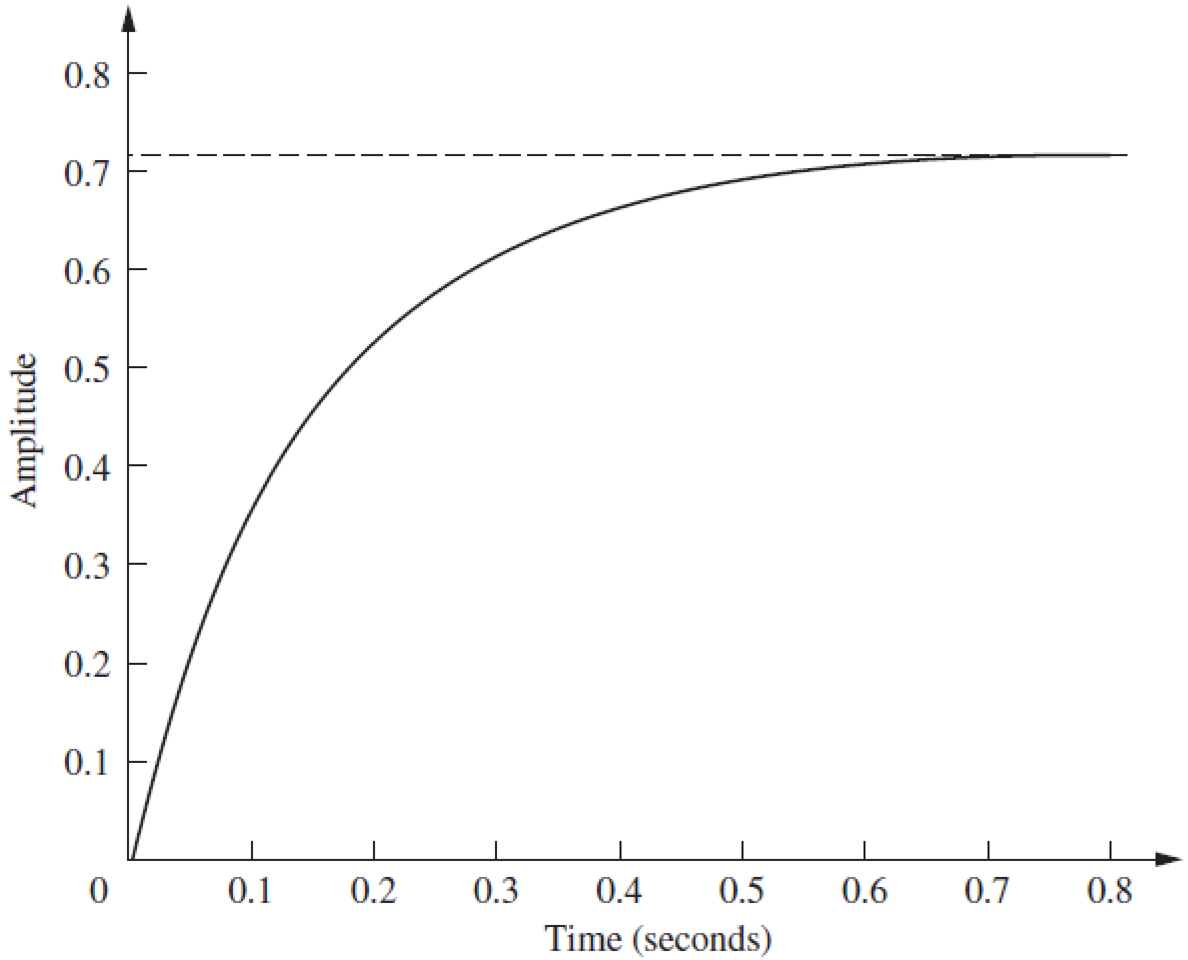

FIGURE: Laboratory results of a system step response test

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General form

- 2 finite poles:

  Complex pole pair determined by the parameters a and b

- No zeros

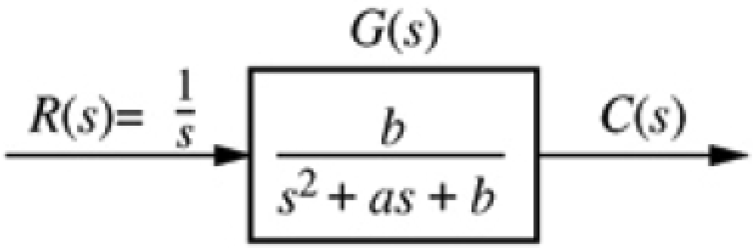
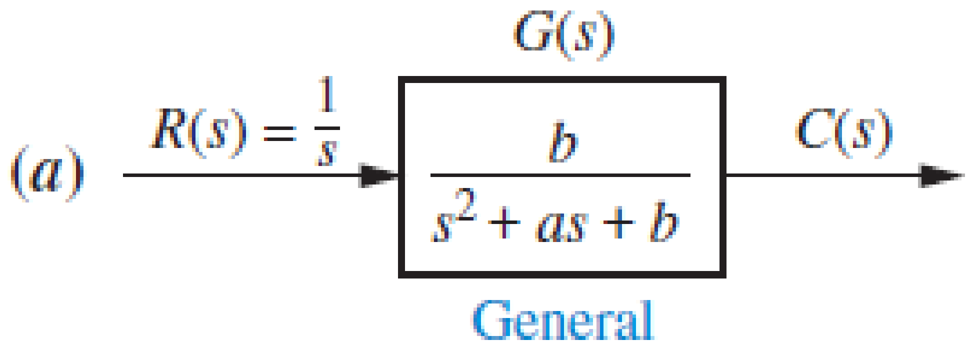
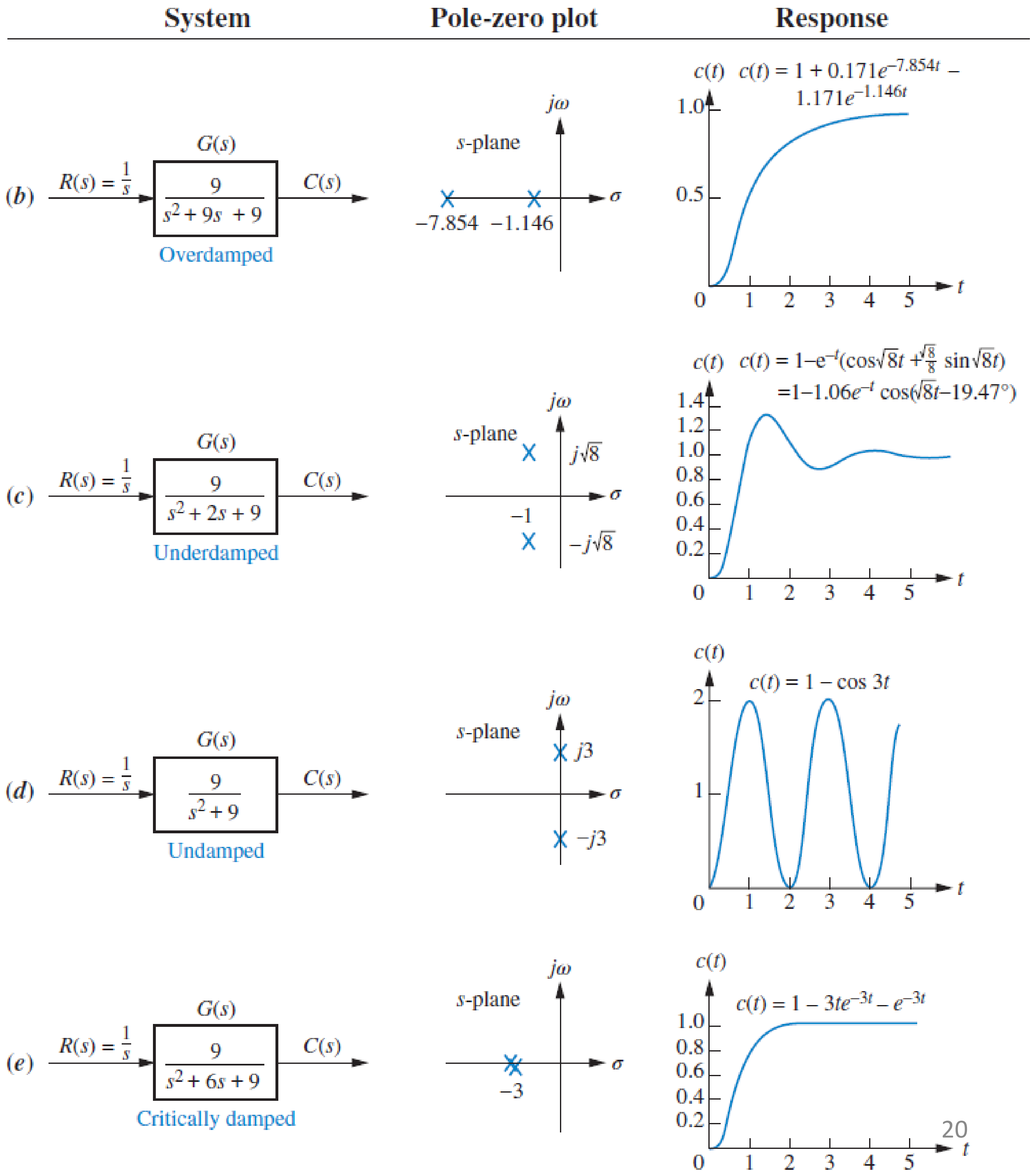


Figure: General 2nd-order system

4.4 Second-order systems: introduction



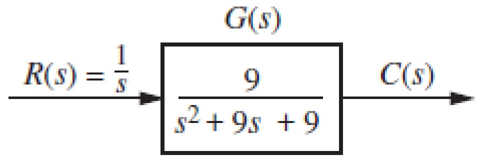


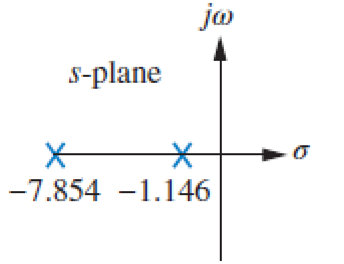
Overdamped response

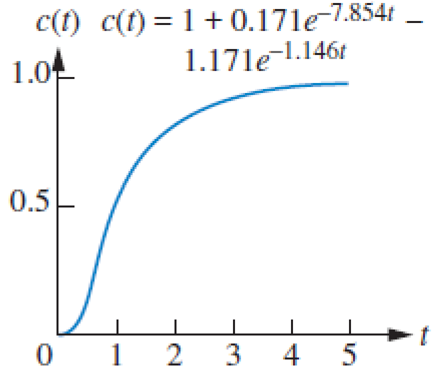
- 1 pole at origin from the unit step input
- System poles: 2 real at σ_1, σ_2
- Natural response: Summation of 2 exponentials

$$c(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

- Exponential frequency: σ_1, σ_2





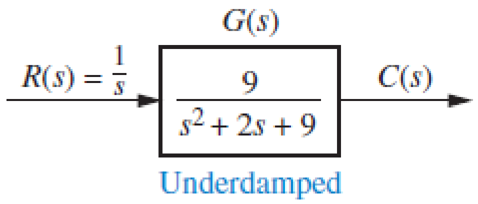


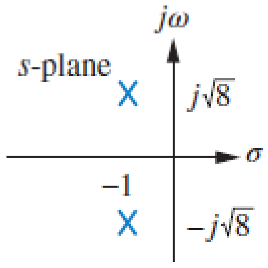
Underdamped response

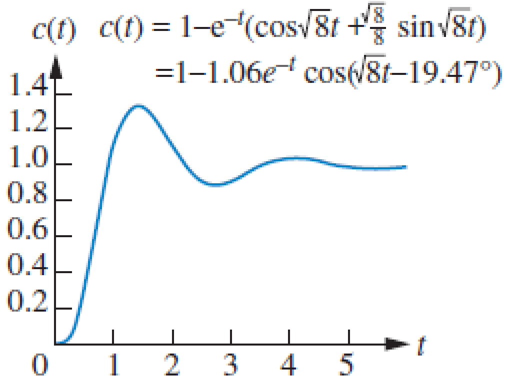
- 1 pole at origin from the unit step input
- System poles: 2 complex at $\sigma_d \pm j\omega_d$
- Natural response: Damped sinusoid with an exponential envelope

$$c(t) = K_1 e^{-\sigma_d t} cos(\omega_d t - \phi)$$

- Exponential decay frequency: σ_d
- Frequency (rad/s): ω_d







Underdamped response characteristics

- Transient response: Exponentially decaying amplitude generated by the real part of the system pole times a sinusoidal waveform generated by the imaginary part of the system pole.
- Damped frequency of oscillation, ω_d: The imaginary part of the system poles.
- Steady state response: Generated by the input pole located at the origin.
- Underdamped response: Approaches a steady state value via a transient response that is a damped oscillation.

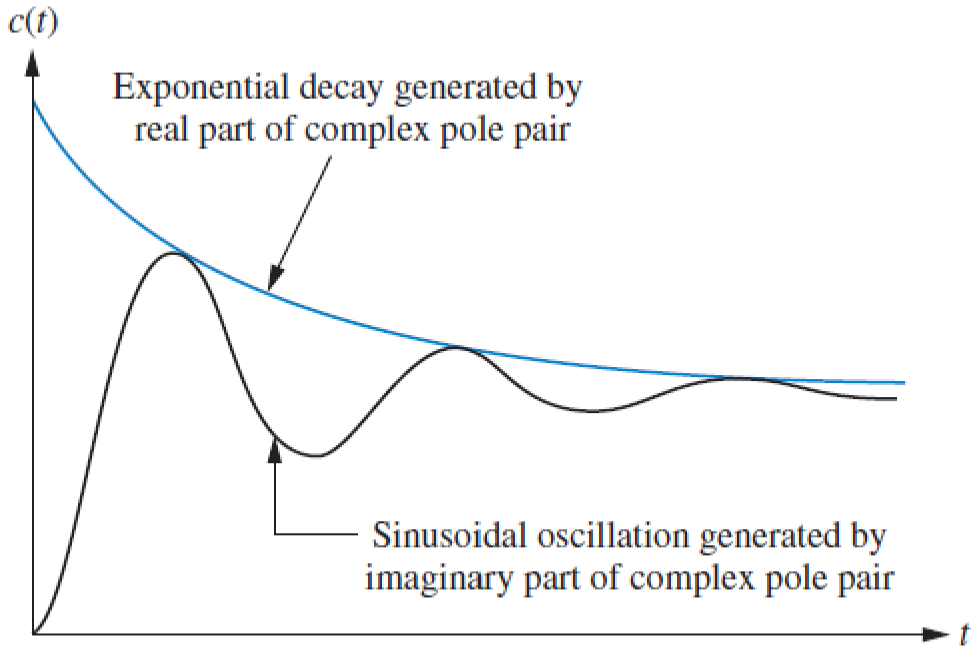


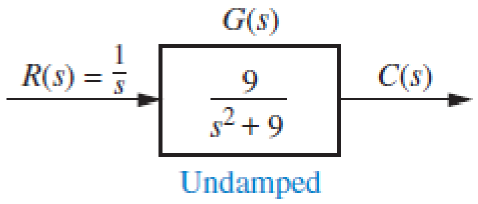
Figure: 2^{nd}-order step response components generated by complex poles

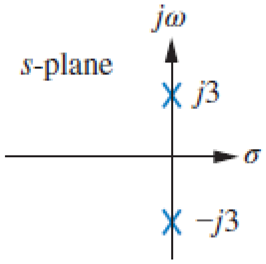
Undamped response

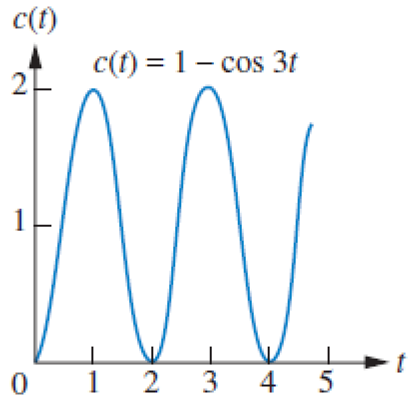
- 1 pole at origin from the unit step input
- System poles: 2 imaginary at $\pm j\omega_1$
- Natural response: Undamped sinusoid

$$c(t) = A\cos\left(\omega_1 t - \phi\right)$$

- Frequency: ω_1





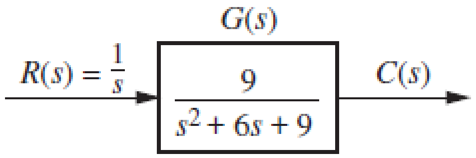


Critically damped response

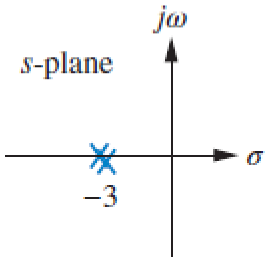
- 1 pole at origin from the unit step input
- System poles: 2 multiple real
- Natural response: Summation of an exponential and a product of time and an exponential

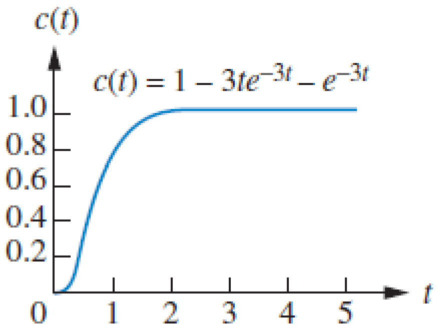
$$c(t) = K_1 e^{-\sigma_1 t} + K_2 t e^{-\sigma_1 t}$$

- Exponential frequency: σ_1
- *Note: Fastest response without overshoot*



Critically damped





Step response damping cases

- ❑Overdamped
- ❑Underdamped
- ❑Undamped
- ❑Critically damped

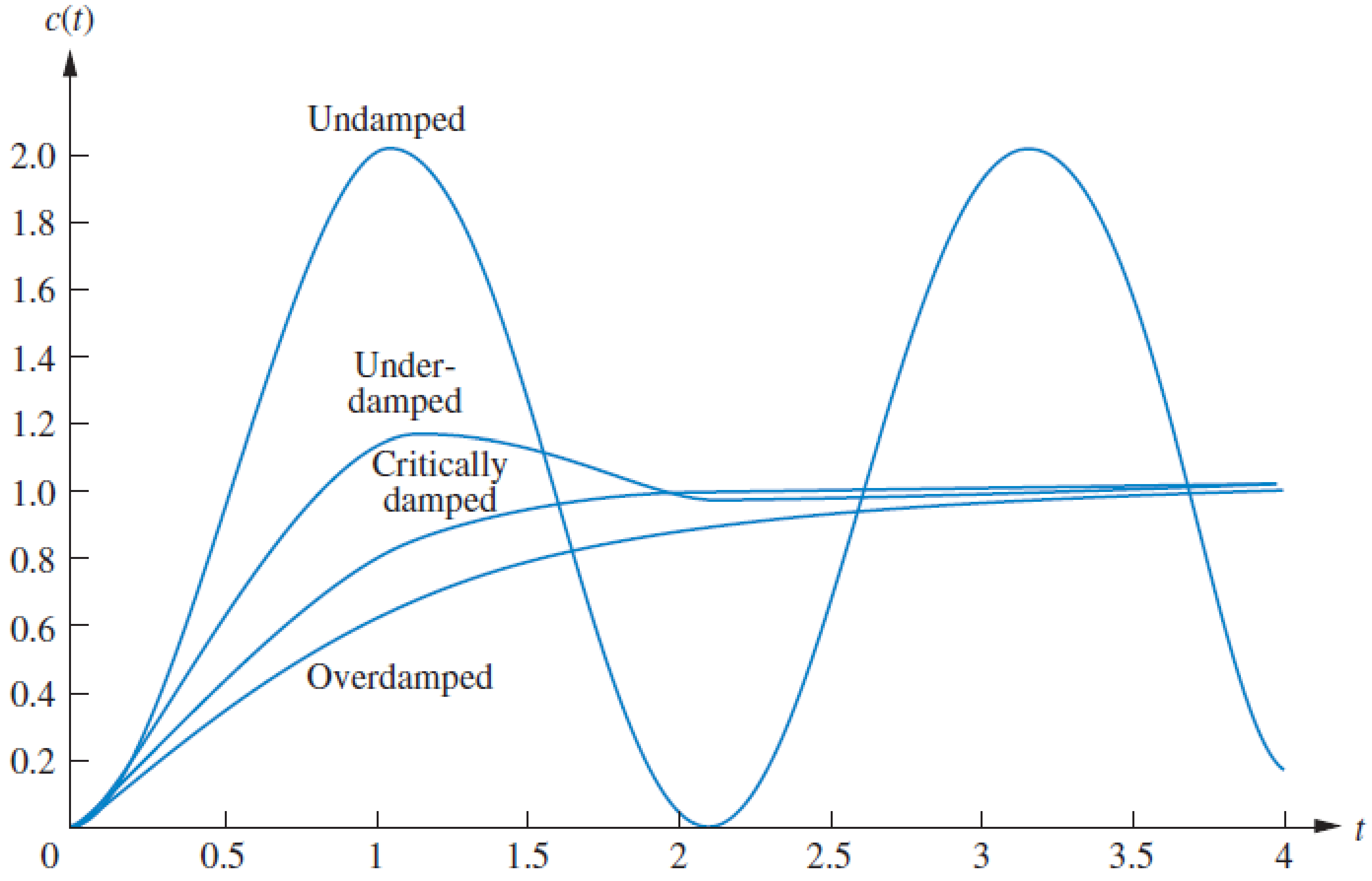


Figure: Step responses for 2nd-order system damping cases

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Specification

- Natural frequency, ω_n
  - The frequency of oscillation of the system without damping
- Damping ratio, ζ

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/s)}} = \frac{1}{2\pi}\frac{\text{Natural period (s)}}{\text{Exponential time constant}}$$

- General TF

$$G(s) = \frac{b}{s^2 + as + b} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

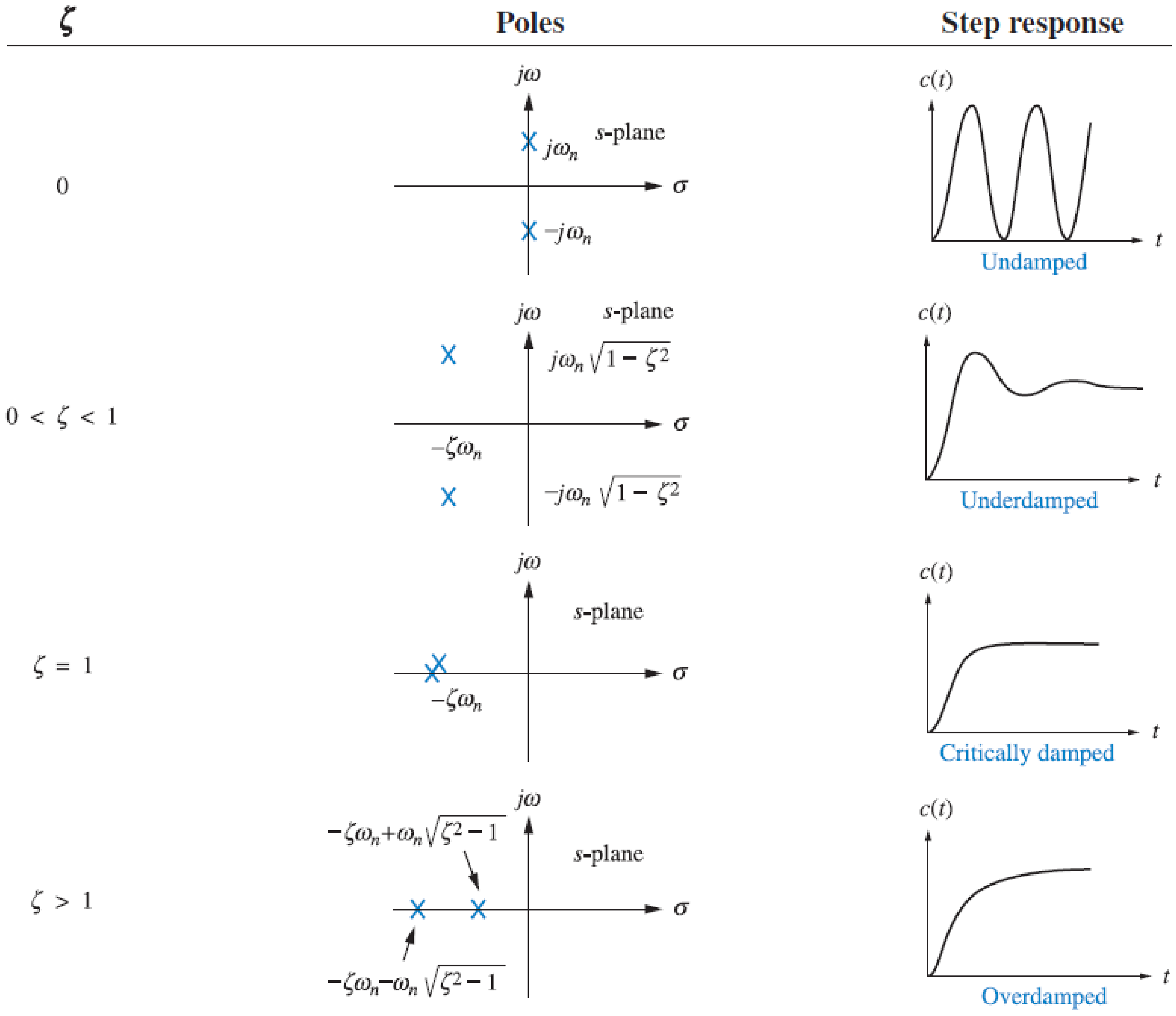
where

$$a = 2\zeta\omega_n, \quad b = \omega_n^2, \quad \zeta = \frac{a}{2\omega_n}, \quad \omega_n = \sqrt{b}$$

Response as a function of ζ

Poles

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$



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Step response

Transfer function

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

...partial fraction expansion...

$$= \frac{1}{s} + \frac{(s + \zeta\omega_n) + \frac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

Step response

Time domain via inverse Laplace transform

$$c(t) = 1 - e^{\zeta\omega_n t}\left(\cos\left(\omega_n\sqrt{1-\zeta^2}\right)t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\right)t\right)$$

...trigonometry & exponential relations...

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_n\sqrt{1-\zeta^2} - \phi)$$

t

where

$$\phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

Responses for ζ values

- Response versus ζ plotted along a time axis normalized to ω_n
    - Lower ζ produce a more oscillatory response
    - ω_n does not affect the nature of the response other than scaling it in time

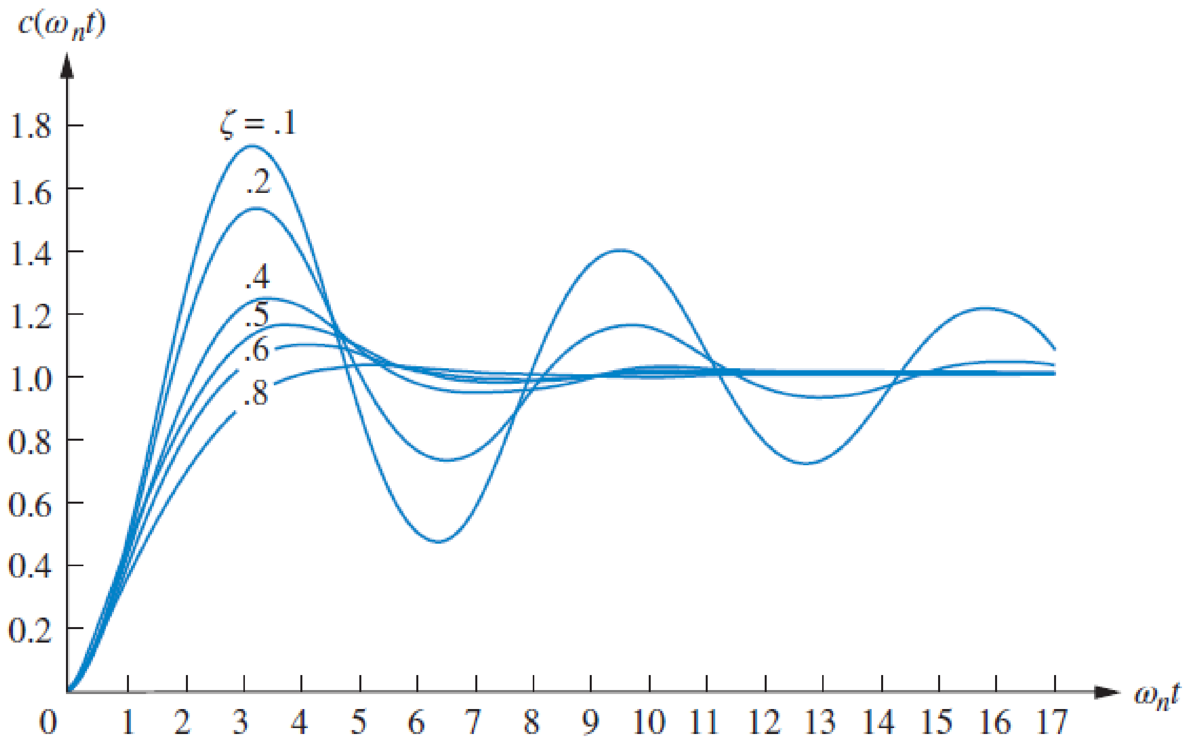
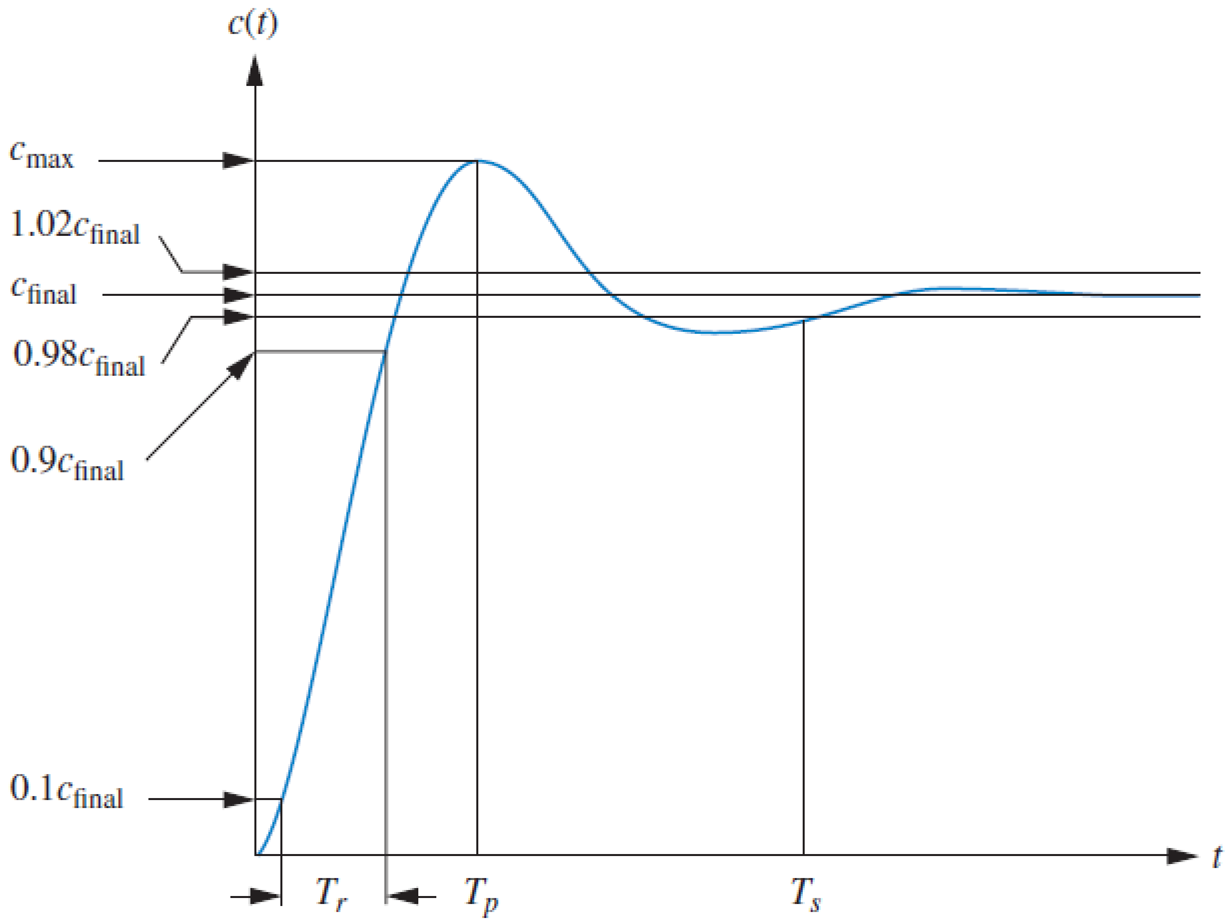


Figure: 2nd-order underdamped responses for damping ratio values

Response specifications

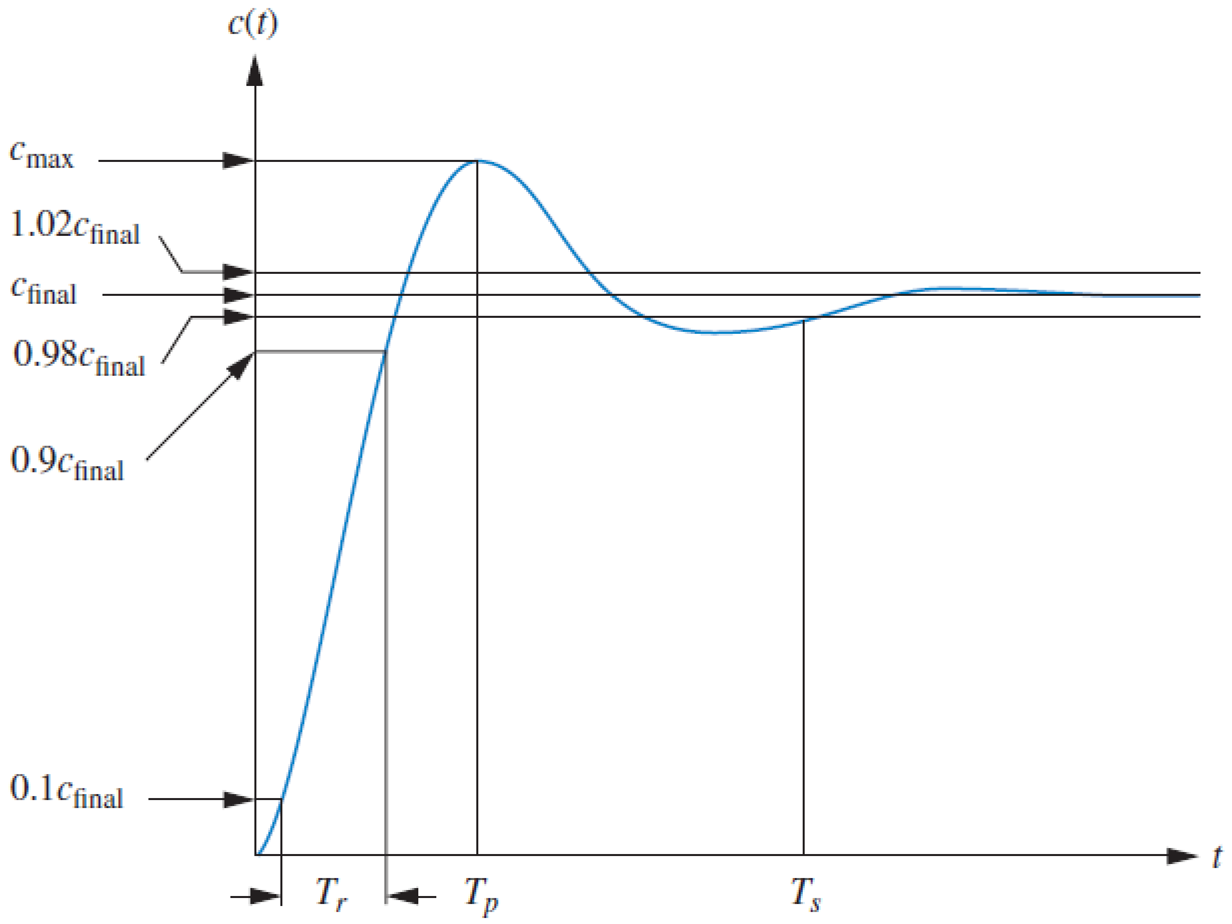
- Rise time, T_r: Time required for the waveform to go from 0.1 of the final value to 0.9 of the final value

- Peak time, T_p: Time required to reach the first, or maximum, peak



Response specifications

- Overshoot, %OS: The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady state value
- Settling time, T_s: Time required for the transient's damped oscillations to reach and stay within ±2% of the steady state value



Evaluation of T_p

- Tp is found by differentiating *c(t)* and finding the zero crossing after t=0, which is simplified by applying a derivative in the frequency domain and assuming zero initial conditions.

$$\mathcal{L}[\dot{c}(t)] = sC(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

...completing the squares in the denominator

...setting the derivative to zero

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Evaluation of *%OS*

$\%OS$ is found by evaluating

$$\%OS = \frac{c_{\text{max}} - c_{\text{final}}}{c_{\text{final}}} \times 100$$

where

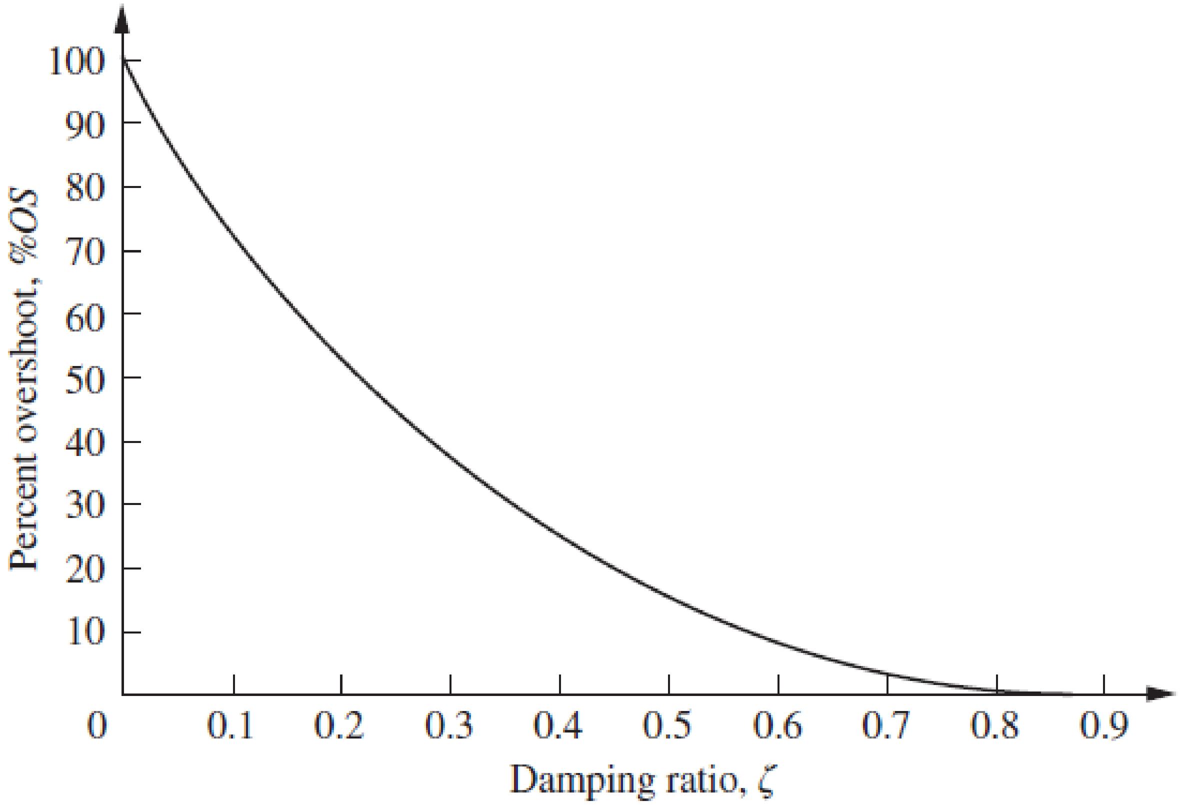
$$c_{\text{max}} = c(T_p), \quad c_{\text{final}} = 1$$

...substitution

$$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

ζ given $\%OS$

$$\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}}$$



Evaluation of T_s

Find the time for which c(t) reaches and stays within ±2% of the steady state value, c_{final}, i.e., the time it takes for the amplitude of the decaying sinusoid to reach 0.02

$$e^{-\zeta\omega_n t}\frac{1}{\sqrt{1-\zeta^2}} = 0.02$$

This equation is a conservative estimate, since we are assuming that

$$\cos(\omega_n\sqrt{1-\zeta^2}t - \phi) = 1$$

Settling time

$$T_s = \frac{-\ln(0.02\sqrt{1-\zeta^2})}{\zeta\omega_n}$$

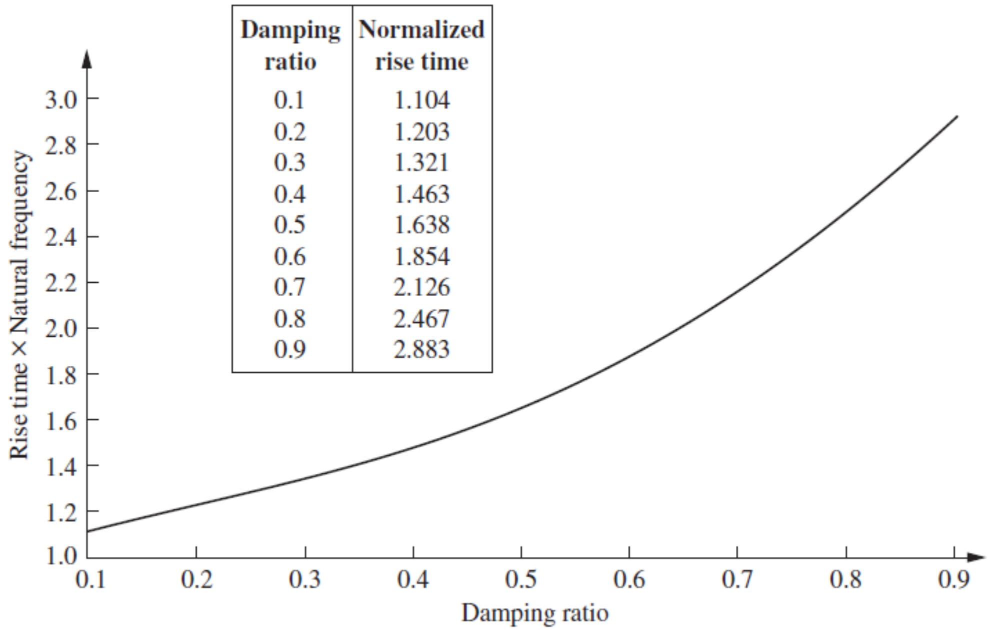
Approximated by

$$T_s = \frac{4}{\zeta\omega_n}$$

Evaluation of T_r

A precise analytical relationship between T_r and ζ cannot be found. However, using a computer, T_r can be found

1. Designate $\omega_n t$ as the normalized time variable
2. Select a value for ζ
3. Solve for the values of $\omega_n t$ that yield c(t) = 0.9 and c(t) = 0.1
4. The normalized rise time $\omega_n T_r$ is the difference between those two values of $\omega_n t$ for that value of ζ



Damping ratio	Normalized rise time
0.1	1.104
0.2	1.203
0.3	1.321
0.4	1.463
0.5	1.638
0.6	1.854
0.7	2.126
0.8	2.467
0.9	2.883

Figure: Normalized T_r vs. ζ for a 2nd-order underdamped response

Location of poles

- *Natural frequency, ω_n* : Radial distance from the origin to the pole
- *Damping ratio, ζ* : Ratio of the magnitude of the real part of the system poles over the natural frequency

$$\cos(\theta) = \frac{-\zeta\omega_n}{\omega_n} = \zeta$$

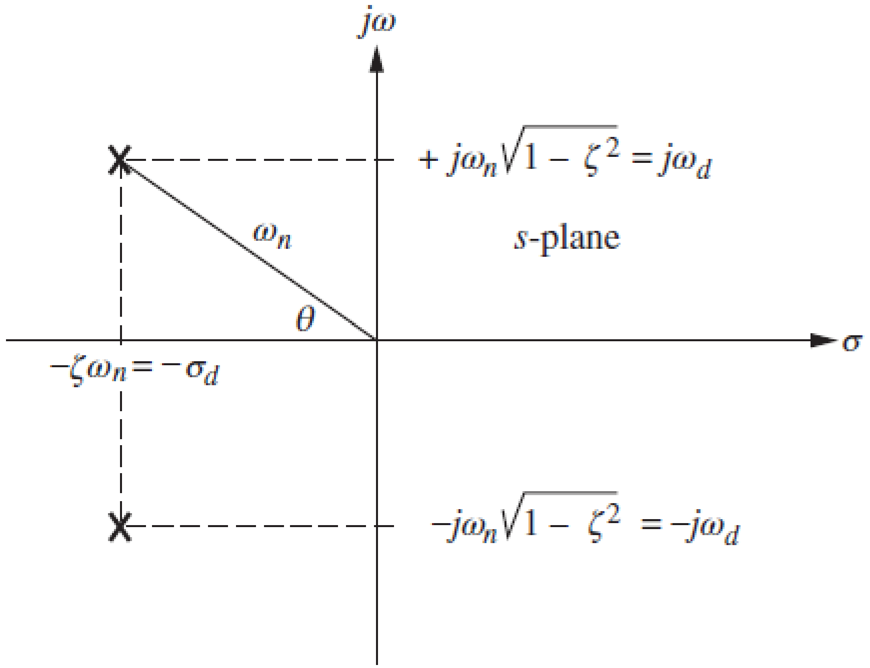


Figure: Pole plot for an underdamped 2nd-order system

Location of poles

- *Damped frequency of oscillation, ω_d* :
  Imaginary part of the system poles

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

- *Exponential damping frequency, σ_d* :
  Magnitude of the real part of the system poles

$$\sigma_d = \zeta\,\omega_n$$

- *Poles*

$$S_{1,2} = -\,\sigma_d \pm j\,\omega_d$$

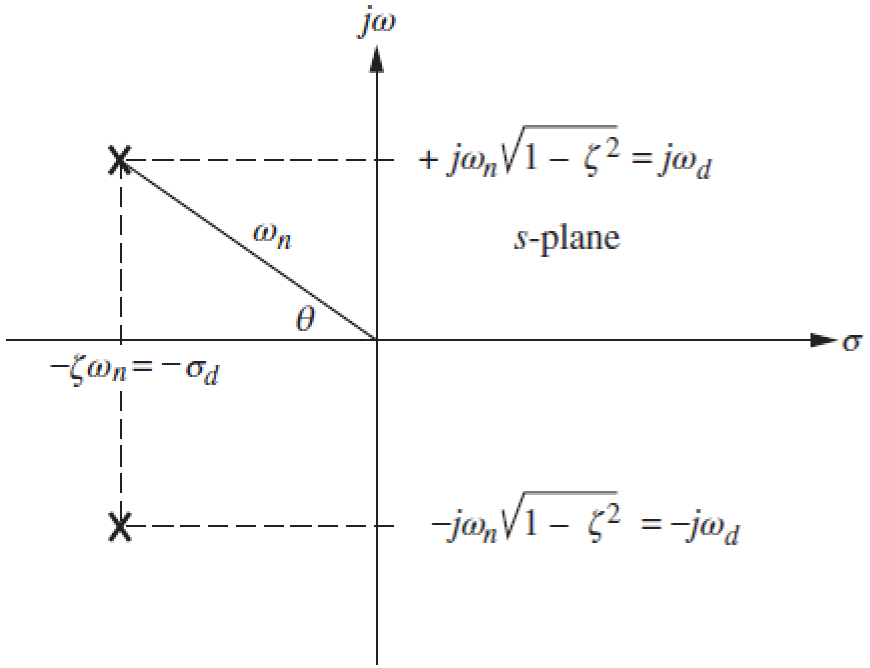


Figure: Pole plot for an underdamped 2nd-order system

Location of poles

- $T_p \propto$ horizontal lines

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

- $T_s \propto$ vertical lines

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma_d}$$

- $\%OS \propto$ radial lines

$$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$\zeta = \cos(\theta)$$

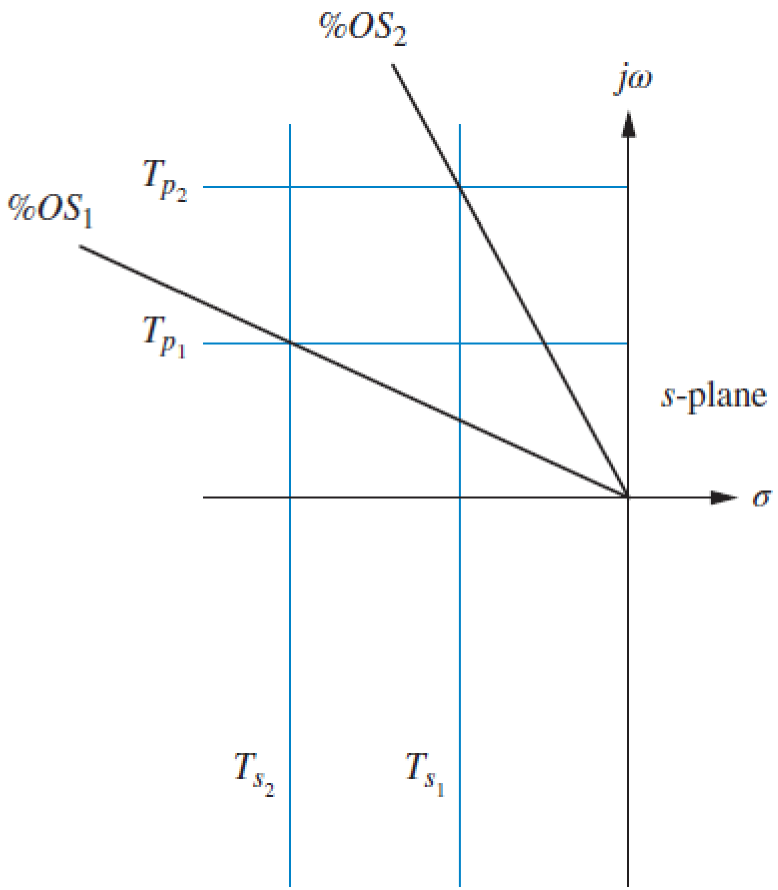


Figure: Lines of constant T_p, T_s, and %OS.

Note: $T_{s2} < T_{s1}$,
$T_{p2} < T_{p1}$,
$\%OS_1 < \%OS_2$.

Underdamped systems

- $T_p \propto$ horizontal lines

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

- $T_s \propto$ vertical lines

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma_d}$$

- %OS $\propto$ radial lines

$$\%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$\zeta = \cos(\theta)$$

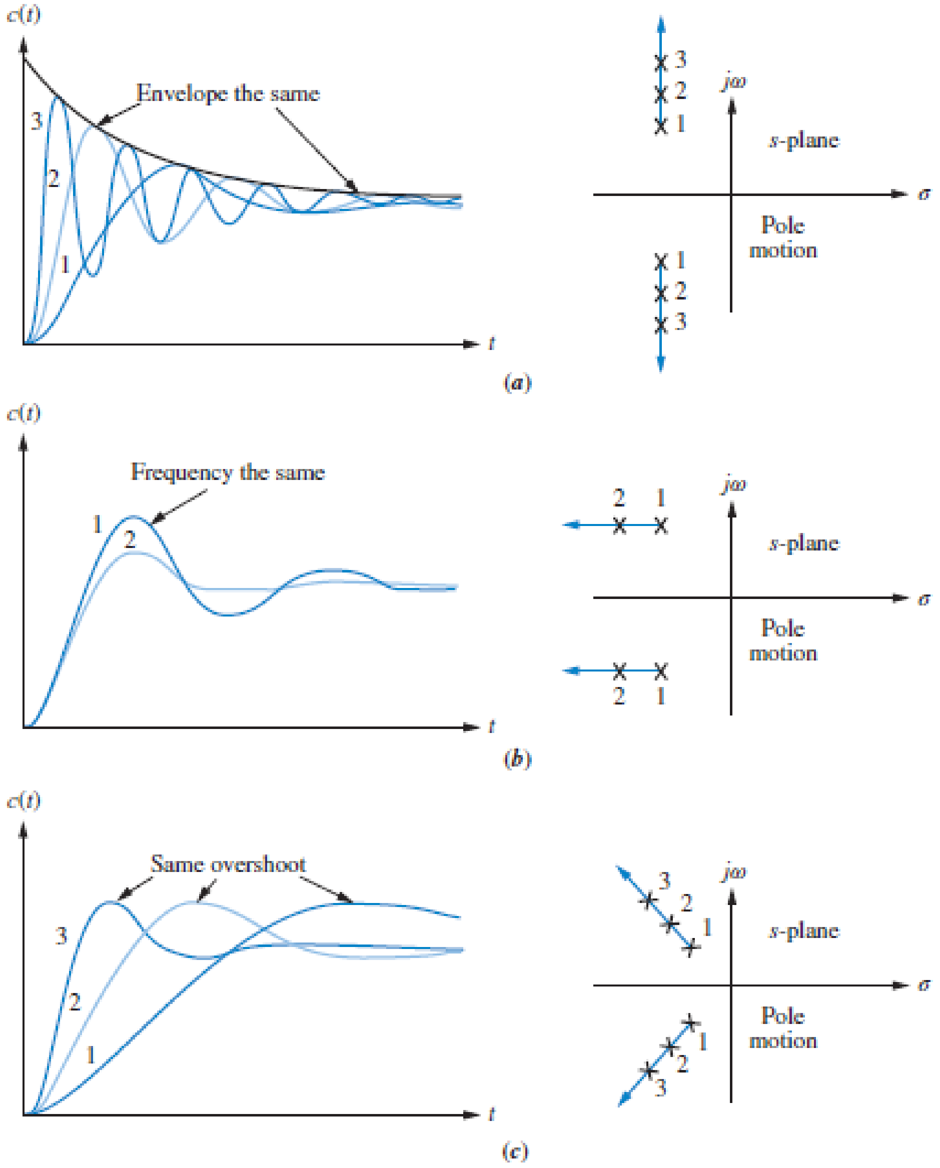


Figure: Step responses of 2nd-order systems as poles move: a. with constant real part,
b. with constant imaginary part,
c. with constant ζ

Chapter outline

4 Time response

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- 4.9 Effects of nonlinearities upon time responses
- 4.10 Laplace transform solution of state equations
- 4.11 Time domain solution of state equations

Effect on the 2nd-order system

- ***Dominant poles:*** The two complex poles that are used to approximate a system with more than two poles as a second-order system
- Conditions: Three pole system with complex poles and a third pole on the real axis

$$s_{1,2} = \zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}, \quad s_3 = -\alpha_r$$

Effect on the 2nd-order system

- Step response of the system in the frequency domain

$$C(s) = \frac{A}{s} + \frac{B(s+\zeta\omega_n) + C\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{D}{s+\alpha_r}$$

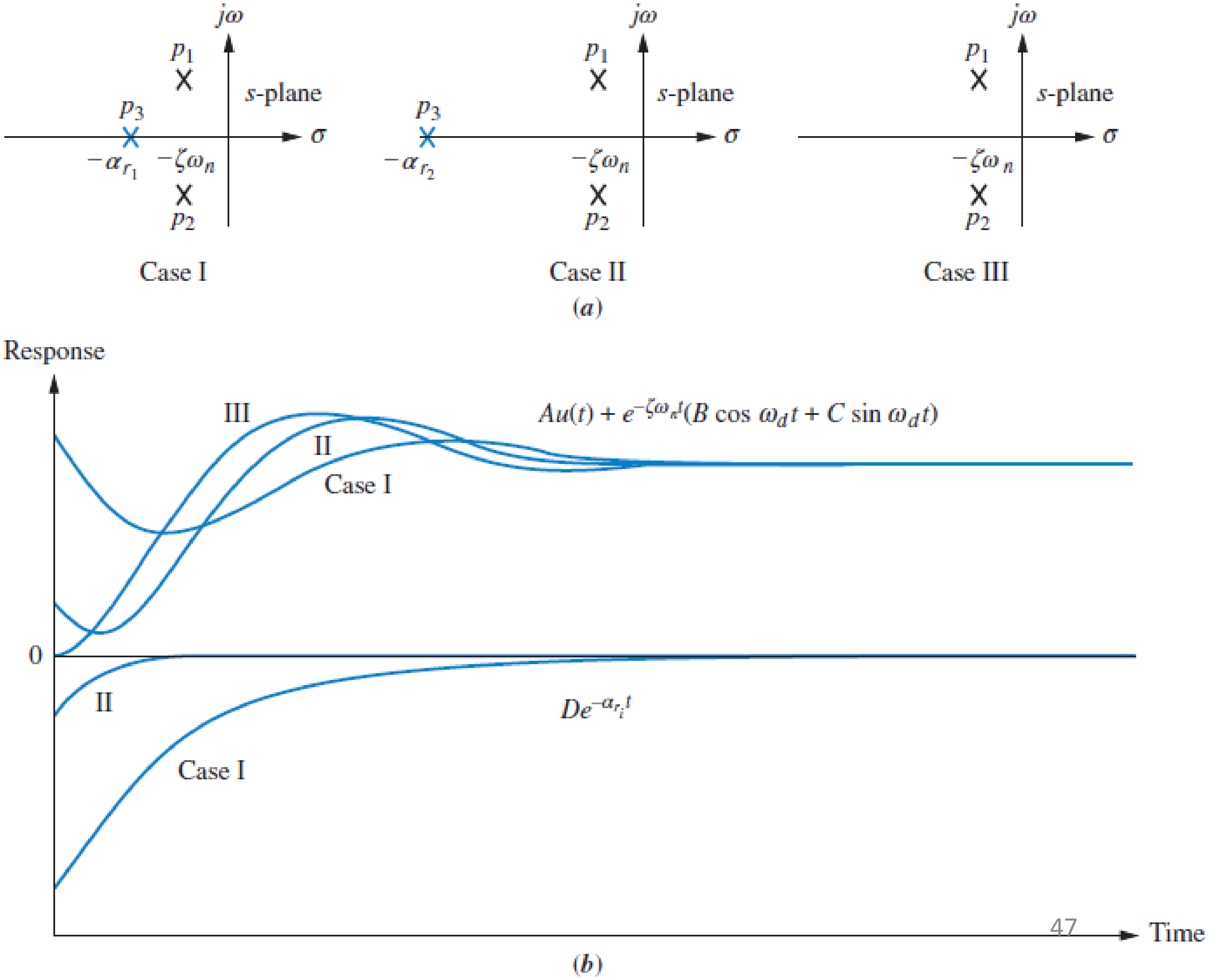
- Step response of the system in the time domain

$$c(t) = Au(t) + e^{-\zeta\omega_n t}(B\cos(\omega_d t) + C\sin(\omega_d t)) + De^{-\alpha_r t}$$

Effect on the 2nd-order system

3 cases for the real pole, $\boldsymbol{\alpha_r}$

1. α_r is not much greater than $\zeta\omega_n$
2. $\alpha_r \gg \zeta\omega_n$
   - Assuming exponential decay is negligible after 5 time constants
   - The real pole is 5× farther to the left than the dominant poles
3. $\alpha_r = \infty$



Effect on the 2nd-order system

- What about the magnitude of the exponential decay?
- Can it be so large that its contribution at the peak time is not negligible?

$$C(s) = \frac{bc}{s(s^2+as+b)(s+c)} = \frac{A}{s} + \frac{Bs+C}{s^2+as+b} + \frac{D}{s+c}$$

- The residue of the third pole, in a three-pole system with dominant second-order poles and no zeros, will actually decrease in magnitude as the third pole is moved farther into the left half-plane.

$$A = 1; \qquad B = \frac{ca-c^2}{c^2+b-ca}$$

$$C = \frac{ca^2-c^2a-bc}{c^2+b-ca}; \quad D = \frac{-b}{c^2+b-ca}$$

Effect on the 2nd-order system

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

$$A = 1; \qquad B = \frac{ca - c^2}{c^2 + b - ca}$$

$$C = \frac{ca^2 - c^2 a - bc}{c^2 + b - ca}; \quad D = \frac{-b}{c^2 + b - ca}$$

As the nondominant pole approaches infinity; or c →∞,
A = 1; B = -1; C = -a; D = 0

Effect on the 2nd-order system

Example 4.8

Comparing Responses of Three-Pole Systems

PROBLEM: Find the step response of each of the transfer functions shown in Eqs. (4.62) through (4.64) and compare them.

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542} \tag{4.62}$$

$$T_2(s) = \frac{245.42}{(s+10)(s^2 + 4s + 24.542)} \tag{4.63}$$

$$T_3(s) = \frac{73.626}{(s+3)(s^2 + 4s + 24.542)} \tag{4.64}$$

Example 4.8

Comparing Responses of Three-Pole Systems

PROBLEM: Find the step response of each of the transfer functions shown in Eqs. (4.62) through (4.64) and compare them.

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542} \quad (4.62)$$

$$T_2(s) = \frac{245.42}{(s+10)(s^2 + 4s + 24.542)} \quad (4.63)$$

$$T_3(s) = \frac{73.626}{(s+3)(s^2 + 4s + 24.542)} \quad (4.64)$$

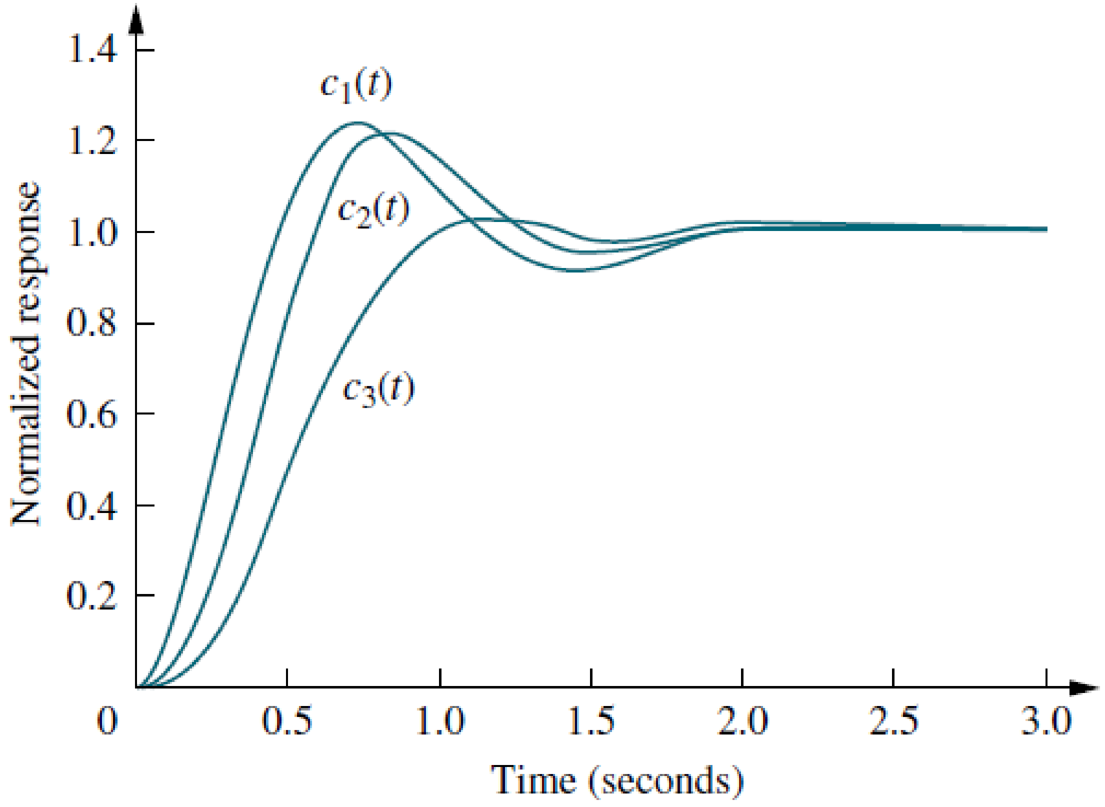
$$c_1(t) = 1 - 1.09e^{-2t}\cos(4.532t - 23.8^\circ)$$

$$c_2(t) = 1 - 0.29e^{-10t} - 1.189e^{-2t}\cos(4.532t - 53.34^\circ)$$

$$c_3(t) = 1 - 1.14e^{-3t} + 0.707e^{-2t}\cos(4.532t + 78.63^\circ)$$

Effect on the 2nd-order system

$$c_1(t) = 1 - 1.09e^{-2t}\cos(4.532t - 23.8°)$$
$$c_2(t) = 1 - 0.29e^{-10t} - 1.189e^{-2t}\cos(4.532t - 53.34°)$$
$$c_3(t) = 1 - 1.14e^{-3t} + 0.707e^{-2t}\cos(4.532t + 78.63°)$$



Effect on the 2nd-order system

PROBLEM: Determine the validity of a second-order approximation for each of these two transfer functions:

a. $G(s) = \dfrac{700}{(s+15)(s^2+4s+100)}$

b. $G(s) = \dfrac{360}{(s+4)(s^2+2s+90)}$

ANSWERS:

a. The second-order approximation is valid.

b. The second-order approximation is not valid.

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Effect on the 2^{nd}-order system

- *Effects on the system response*
  - Residue, or amplitude
  - Not the nature, e.g., exponential, damped sinusoid, etc.
  - Greater as the zero approaches the dominant poles
- *Conditions:* Real axis zero added to a two-pole system

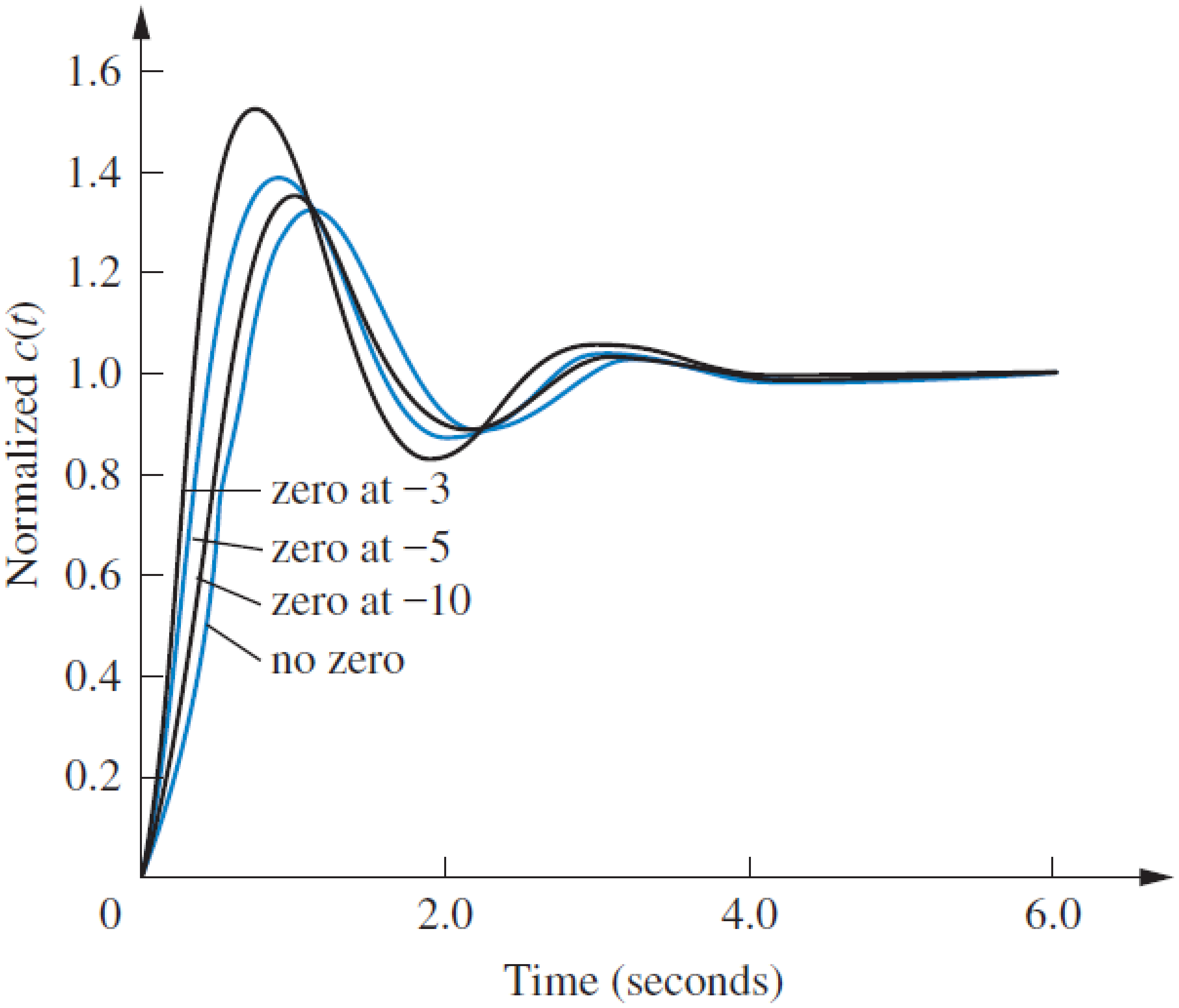


Figure: Effect of adding a zero to a 2-pole system

Effect on the 2nd-order system

Assume a group of poles and a zero far from the poles.

...partial-fraction expansion...

$$\begin{aligned} T(s) &= \frac{s+a}{(s+b)(s+c)} \\ &= \frac{A}{s+b} + \frac{B}{s+c} \\ &= \frac{(-b+a)/(-b+c)}{s+b} + \frac{(-c+a)/(-c+b)}{s+c} \end{aligned}$$

If the zero is far from the poles, then a ≫ b and a ≫ c, and

$$\begin{aligned} T(s) &\approx a\left\{\frac{1/(-b+c)}{s+b} + \frac{1/(-c+b)}{s+c}\right\} \\ &= \frac{a}{(s+b)(s+c)} \end{aligned}$$

Zero looks like a ***simple gain factor*** and does not change the relative amplitudes of the components of the response.

Effect on the 2^{nd}-order system

Another view...

- Response of the system, C(s)
- System TF, T(s)
- Add a zero to the system TF, yielding, (s + a)T(s)
- Laplace transform of the response of the system

$$(s + a)C(s) = sC(s) + aC(s)$$

- Response of the system consists of 2 parts
  - The derivative of the original response
  - A scaled version of the original response

Effect on the 2^{nd}-order system

3 cases for ***a***

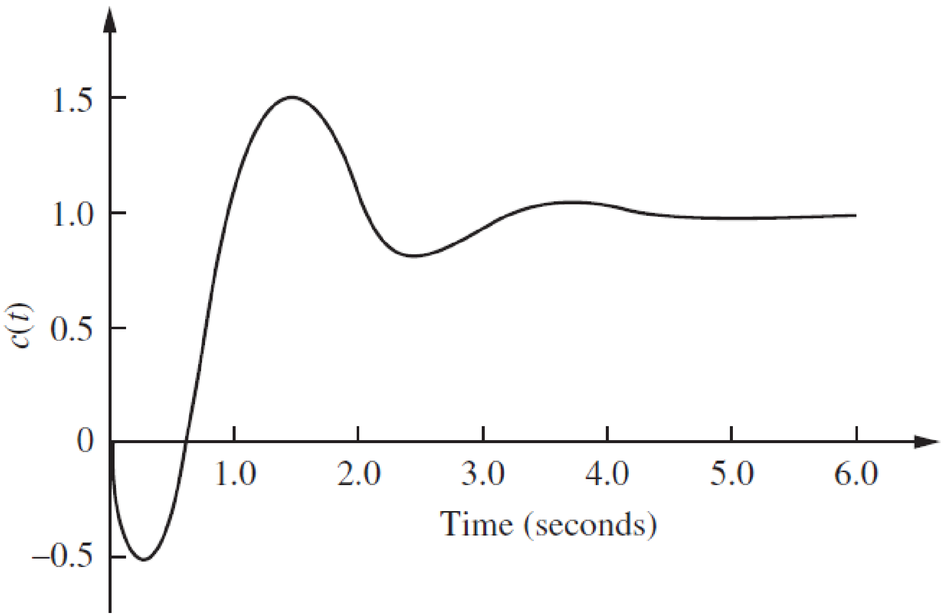
- a is very large
  - Response → aC(s), a scaled version of the original response
- a is not very large
  - Response has additional derivative component producing more overshoot
- a is negative – right-half plane zero
  - Response has additional derivative component with an opposite sign from the scaled response term

Non-minimum-phase system,

- Non-minimum-phase system:

System that is causal and stable whose inverses are causal and unstable.

  - Characteristics: If the derivative term, sC(s), is larger than the scaled response, aC(s), the response will initially follow the derivative in the opposite direction from the scaled response.



$$T(s) = \frac{K(s+z)}{(s+p_3)\,(s^2+as+b)}$$

Example 4.10

Evaluating Pole-Zero Cancellation Using Residues

PROBLEM: For each of the response functions in Eqs. (4.86) and (4.87), determine whether there is cancellation between the zero and the pole closest to the zero. For any function for which pole-zero cancellation is valid, find the approximate response.

$$C_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} \tag{4.86}$$

$$C_2(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} \tag{4.87}$$

SOLUTION: The partial-fraction expansion of Eq. (4.86) is

$$C_1(s) = \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5} \tag{4.88}$$

The residue of the pole at -3.5, which is closest to the zero at -4, is equal to 1 and is not negligible compared to the other residues. Thus, a second-order step response approximation cannot be made for $C_1(s)$. The partial-fraction expansion for $C_2(s)$ is

$$C_2(s) = \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} + \frac{0.033}{s+4.01} \tag{4.89}$$

The residue of the pole at -4.01, which is closest to the zero at -4, is equal to 0.033, about two orders of magnitude below any of the other residues. Hence, we make a second-order approximation by neglecting the response generated by the pole at -4.01:

$$C_2(s) \approx \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} \tag{4.90}$$

and the response $c_2(t)$ is approximately

$$c_2(t) \approx 0.87 - 5.3e^{-5t} + 4.4e^{-6t} \tag{4.91}$$

Try at home

30. For the following response functions, determine if pole-zero cancellation can be approximated. If it can, find percent overshoot, settling time, rise time, and peak time. [Section: 4.8].

a. $C(s) = \dfrac{(s+3)}{s(s+2)(s^2+3s+10)}$

b. $C(s) = \dfrac{(s+2.5)}{s(s+2)(s^2+4s+20)}$

c. $C(s) = \dfrac{(s+2.1)}{s(s+2)(s^2+s+5)}$

d. $C(s) = \dfrac{(s+2.01)}{s(s+2)(s^2+5s+20)}$